B. Ozgener, S. Cavdar, H. A. Ozgener

The Extension of the 2-D Finite Element/Boundary Element Hybrid Method to General Multigroup Neutron Diffusion Theory

Abstract

The finite element-boundary element hybrid method developed previously for reflected systems and restricted to one or two group neutron diffusion theory has been extended to the general multigroup neutron diffusion theory by using the boundary integral equation of multigroup neutron diffusion theory. A linear or bilinear 2-D FEM formulation in the core combined with a 2-D linear BEM formulation in the reflector constitute the basic discretization procedure. Use of the boundary integral equation of multigroup diffusion theory transforms all group-to-group scattering domain integrals into surface integrals in the reflector. Hence the need for a reflector domain mesh is completely eliminated. Via comparisons with pure FEM and BEM solutions of the reflected systems within the context of three and four group diffusion theories, the present formulation is validated and assessed.

Die Erweiterung der zwei-dimensionalen Finite Elemente/Randelemente Hybridmethode zum allgemeinen Mehrgruppen Neutrondiffusionstheorie

Abstrakt

Die Finite Elemente-Randelemente Hybridmethode wurde für reflektierte Systeme entwickelt und blieb auf Ein- oder Zweigruppen Neutrondiffusionstheorie beschränkt. Diese Arbeit erweitert die obengenannte Methode mit Hilfe der Randintegralgleichung der Mehrgruppendiffusionstheorie zum Mehrgruppendiffusiongleichungen. Eine lineare oder bilineare zweidimensionale Finite Elemente Formulierung für den Reaktorkern, die mit einer linearen zwei-dimensionale Randelemente Formulierung für den Reflektor kombiniert wird, stellt das grundlegende Diskretisierungverfahren dar. Da die Benutzung der Randintegralgleichung der Mehrgruppendiffusionstheorie alle Streuungvolumenintegrale zur Oberflächeintegrale in dem Reflektor transformiert, wird eine Reflektorvolumenmasche überflüssig. Durch Vergleiche mit reinen Finite Elemente und Randelemente Lösungen für die reflektierte Systeme in Kontext der Drei- und Viergruppen Theorien wird die präsentierte Formulierung validiert und bewertet.

1. Introduction

The first applications of the finite element method (FEM) and the boundary element method (BEM) date back to more than three [1] and almost two [2] decades ago respectively. The sparse and positive definite nature of the resulting coefficient matrices and the ease of treatment of irregular geometries constitute the basic advantages of the FEM in its application to the neutron diffusion and transport equations and is discussed fully elsewhere [3]. On the other hand, FEM requires a domain mesh and the number of unknowns could dramatically increase with mesh refinement. This fact constitutes the basic disadvantage of FEM. The BEM, on the other hand, has the distinct advantage of confining the unknowns to the boundaries of each homogeneous region and the resultant dramatic decrease on the unknowns relative to the alternative methods (i.e. FEM) [4]. On the other hand, the full and nonsymmetric nature of the submatrices corresponding to each homogeneous region is its basic disadvantage [5]. The first application of the BEM to the multiregion neutron diffusion equation is quite dates back to less than a decade ago [6].

In the application of the BEM to the multiregion problems of neutron diffusion, two distinct approaches have been taken. In one approach, which may be called the classical BEM approach, the BEM equations for each of the homogeneous regions in the system are assembled together in a block matrix form using the concept of the "virtual side" and continuity of the flux and current across material interfaces [7]. In the second approach which is based on the domain decomposition method and could be referred to as domain decomposition BEM, the classical fission source iteration procedure of criticality problems is eliminated. There are two variants of the domain decomposition BEM. The first variant is called the hierarchical domain decomposition BEM [8],[9],[10]. The second variant of the domain decomposition BEM is called the response matrix BEM [11]. A short description of the variants of domain decomposition BEM is given elsewhere [12]. In a recent publication, the response matrix BEM is based on the application of the Galerkin method and quite accurate results are obtained [13].

The first application of a hybrid FEM/BEM formulation to the neutron diffusion equation is quite recent [12]. In that work, a FEM formulation in the core is combined with a BEM approximation in the reflector, for 2-D systems, but the number of groups was limited at most to two. In this work, we extend this hybrid FEM/BEM approach to general multigroup diffusion theory with arbitrary number of groups, using the boundary integral equation of multigroup diffusion theory [14]. The merit of our proposed formulation will be assessed via comparisons with pure BEM and FEM solutions within the context of three and four group diffusion theories.

2. Theory

We consider a two-dimensional nuclear system consisting of a reactor core (C) and a reflector (R). The volume of the core and the reflector are denoted as VC and VR respectively. We denote the outer surface of the core and reflector by SC and SR respectively. Either vacuum (v) or reflective (r) boundary condition prevails at the outer surface. Thus: $S^C = S_v^C \cup S_r^C$ and $S^R = S_v^R \cup S_r^R$. The core and the reflector are joined by the interface SI (see Figure 1).

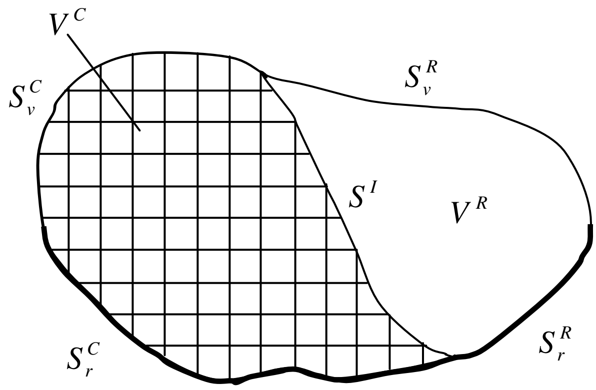

Figure 1 Reflected system

For ease of presentation of the current formulation, we would like extend the concept of the virtual side which was introduced for the multiregion boundary element formulation [7]. In that work, a virtual side was defined "as a union of linear segments either joining a pair of neighboring cells or belonging solely to a distinct cell at system outer boundary". Our system in this case consists of two cells: the core and the reflector. The reflector cell has two virtual sides: The core-reflector interface (I) and its outer surface (R) which is a combined boundary virtual side. To render the application of the concept of virtual side to the core cell, to which we'll apply the finite element method (FEM), possible, we define the union of the interior and external boundary (V/SI) of the core also as a virtual side (C). Thus, the core consists of two virtual sides: core virtual side (C) and core-reflector interface (I) virtual sides. Thus our two-cell system consists of three virtual sides: C (core except the interface), I (interface), R (reflector outer surface).

We apply the linear or bilinear FEM to the core cell using the Galerkin approach to get [12]

$$\underline{\underline{A}}_{g,C}^{C,C}\,\underline{u}_g^C + \underline{\underline{A}}_{g,C}^{C,I}\,\underline{u}_g^I = \underline{s}_{g,C}^C \tag{1}$$

$$\underline{\underline{A}}_{g,C}^{I,C}\,\underline{u}_g^C + \underline{\underline{A}}_{g,C}^{I,I}\,\underline{u}_g^I = \underline{s}_{g,C}^I \tag{2}$$

in the virtual side notation [7]

Partitioned vectors and matrices of equations (1) and (2), which follow the virtual side notation, are defined in terms of vectors and matrices of our previous work [12] in equations (3) to (11). Detailed definition of the quantities an the right hand sides of equations (3)-(11) is given in that work and will not be repeated here since increasing the number of groups from two to an arbitrary number does not result in any modification in their definitions.

$$\underline{u}_g^C = \underline{\phi}_g^C \tag{3}$$

$$\underline{u}_g^I = \begin{bmatrix} \underline{\phi}_g^I \\ \underline{J}_g^I \end{bmatrix} \tag{4}$$

$$\underline{\underline{A}}_{g,C}^{C,C} = \underline{\underline{A}}_g^C \tag{5}$$

$$\underline{\underline{A}}_{g,C}^{C,I} = \begin{bmatrix} \underline{\underline{A}}_g^{CI} & \underline{\underline{0}} \end{bmatrix} \tag{6}$$

$$\underline{\underline{A}}_{g,C}^{I,C} = \begin{bmatrix} \left(\underline{\underline{A}}_g^{CI}\right)^T \\ \underline{\underline{0}} \end{bmatrix} \tag{7}$$

$$\underline{\underline{A}}_{g,C}^{I,I} = \begin{bmatrix} \underline{\underline{A}}_g^I & \underline{\underline{A}}_g^{IJ} \\ \underline{\underline{0}} & \underline{\underline{0}} \end{bmatrix} \tag{8}$$

$$\underline{s}_{g,C}^C = \sum_{g'=1}^{g-1} \left[\underline{\underline{S}}_{g\leftarrow g'}^C \underline{\phi}_{g'}^C + \underline{\underline{S}}_{g\leftarrow g'}^{CI} \underline{\phi}_{g'}^I \right] + \frac{1}{k} \sum_{g'=1}^{G} \left[\underline{\underline{F}}_{g\leftarrow g'}^C \underline{\phi}_{g'}^C + \underline{\underline{F}}_{g\leftarrow g'}^{CI} \underline{\phi}_{g'}^I \right] \tag{9}$$

$$\underline{s}_{g,C}^I = \begin{bmatrix} \underline{q}_{g,C}^I \\ \underline{0} \end{bmatrix} \tag{10}$$

$$\underline{q}_{g,C}^I = \sum_{g'=1}^{g-1} \left[\left(\underline{\underline{S}}_{g\leftarrow g'}^{CI}\right)^T \underline{\phi}_{g'}^C + \underline{\underline{S}}_{g\leftarrow g'}^I \underline{\phi}_{g'}^I \right] + \frac{1}{k} \sum_{g'=1}^{G} \left[\left(\underline{\underline{F}}_{g\leftarrow g'}^{CI}\right)^T \underline{\phi}_{g'}^C + \underline{\underline{F}}_{g\leftarrow g'}^I \underline{\phi}_{g'}^I \right] \tag{11}$$

The application of the boundary element method (BEM) to the reflector is based on the within-group integral equation (equation (27) in [15]) taking the absence of the fission source in the reflector into account:

$$c(\vec{\rho})\phi_g(\vec{\rho}) + \int_{S_R} \frac{G_g^R(\vec{r},\vec{\rho})}{D_g^R} J_g(\vec{r})dS + \int_{S_V} \frac{\partial G_g^R}{\partial n}(\vec{r},\vec{\rho})\phi_g(\vec{r})dS$$
$$= \sum_{g'=1}^{g-1} s_{g,g'}^R \left[c(\vec{\rho})\phi_{g'}(\vec{\rho}) + \int_{S_R} \frac{G_g^R(\vec{r},\vec{\rho})}{D_{g'}^R} J_{g'}(\vec{r})dS + \int_{S_R} \frac{\partial G_g^R}{\partial n}(\vec{r},\vec{\rho})\phi_{g'}(\vec{r})dS \right] \quad (12)$$

Here $S_R = S^R \cup S^I$ and encompasses the whole boundary of the reflector, including the core-reflector interface. We assume that the boundary SR of the reflector is divided into NR linear boundary elements, that is:

$$\phi_g(\vec{r}) = \sum_{i=1}^{N_R} \psi_i(\vec{r})\phi_g^i \quad (13)$$

$$J_g(\vec{r}) = \sum_{i=1}^{N_R} \psi_i(\vec{r}) J_g^i \quad (14)$$

Here $\psi_i(\vec{r})$ is the linear trial function of node *i* of the BEM mesh of the reflector. $\psi_i(\vec{r})$ is linear in the adjacent boundary elements to which the node *i* belongs. On other elements $\psi_i(\vec{r})$ vanishes. That is:

$$\psi_i(\vec{r}) = \delta_{ij}, \quad \begin{matrix} i = 1,\ldots,N_R \\ j = 1,\ldots,N_R \end{matrix} \quad (15)$$

We apply point collocation by demanding (12) to be valid for $\vec{\rho} = \vec{\rho}_i \ (i = 1,\ldots,N_R)$ when the approximations (13) and (14) are made and obtain the matricial equation:

$$\underline{\underline{H}}_g^R \, \underline{\phi}_g^R + \underline{\underline{G}}_g^R \, \underline{J}_g^R = \underline{q}_g^R \quad (16)$$

where:

$$\left(h_g^R\right)_{ij} = c(\vec{\rho}_i)\delta_{ij} + \int_{S_R} \frac{\partial G_g^R}{\partial n}(\vec{r},\vec{\rho}_i)\psi_j(\vec{r})dS \quad (17)$$

$$\left(g_g^R\right)_{ij} = \int_{S_R} \frac{G_g^R(\vec{r},\vec{\rho}_i)}{D_g^R} \psi_j(\vec{r})dS \quad (18)$$

with

$$\underline{q}_g^R = \underline{\underline{H}}_g^R \, \underline{d}_g^R + \underline{\underline{G}}_g^R \, \underline{e}_g^R \quad (19)$$

where:

$$\underline{d}_g^R = \sum_{g'=1}^{g-1} s_{g,g'} \underline{\phi}_{g'}^R \quad (20)$$

$$\underline{e}_g^R = \sum_{g'=1}^{g-1} \frac{s_{g,g'} D_g^R}{D_{g'}^R} \underline{J}_{g'}^R \tag{21}$$

The relevant boundary conditions are:

$$J_g(\vec{r}) = 0, \quad \vec{r} \in S_r^R \tag{22}$$

$$\phi_g(\vec{r}) = 2J_g(\vec{r}), \quad \vec{r} \in S_v^R \tag{23}$$

When (22) and (23) are applied and partitioning into virtual sides are carried out, (16) can be rewritten in the virtual side notation [7]:

$$\underline{\underline{A}}_{g,R}^{I,I} \underline{u}_g^I + \underline{\underline{A}}_{g,R}^{I,R} \underline{u}_g^R = \underline{s}_{g,R}^I \tag{24}$$

$$\underline{\underline{A}}_{g,R}^{R,I} \underline{u}_g^I + \underline{\underline{A}}_{g,R}^{R,R} \underline{u}_g^R = \underline{s}_{g,R}^R \tag{25}$$

where:

$$\underline{u}_g^R = \begin{bmatrix} \underline{\phi}_g^{R(r)} \\ \underline{J}_g^{R(v)} \end{bmatrix} \tag{26}$$

$\underline{\phi}_g^{R(r)}$ is the vector of nodal flux values on the virtual side R at which the reflective boundary condition applies while $\underline{J}_g^{R(r)}$ is the vector of nodal current values on the virtual side R at when the vacuum boundary condition applies. The definitions of the partitioned vectors and matrices of (24) and (25) follow again the virtual side notation as:

$$\underline{\underline{A}}_{g,R}^{I,I} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{H}}_{g,R}^{I,I} & -\underline{\underline{G}}_{g,R}^{I,I} \end{bmatrix} \tag{27}$$

$$\underline{\underline{A}}_{g,R}^{I,R} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{H}}_{g,R}^{I,R(r)} & 2\underline{\underline{H}}_{g,R}^{I,R(v)} + \underline{\underline{G}}_{g,R}^{R(v)} \end{bmatrix} \tag{28}$$

$$\underline{\underline{A}}_{g,R}^{R,I} = \begin{bmatrix} \underline{\underline{H}}_{g,R}^{R,I} & -\underline{\underline{G}}_{g,R}^{R,I} \end{bmatrix} \tag{29}$$

$$\underline{\underline{A}}_{g,R}^{R,R} = \begin{bmatrix} \underline{\underline{H}}_{g,R}^{R,R(r)} & 2\underline{\underline{H}}_{g,R}^{R,R(v)} + \underline{\underline{G}}_{g,R}^{R,R(v)} \end{bmatrix} \tag{30}$$

$$\underline{s}_{g,R}^I = \begin{bmatrix} \underline{0} \\ \underline{q}_{g,R}^I \end{bmatrix} \tag{31}$$

$$\underline{s}_{g,R}^R = \underline{q}_{g,R}^R \tag{32}$$

Assembly process is carried out combining the core FEM equations, (1) and (2), with the reflector BEM equations, (24) and (25), yielding:

$$\begin{bmatrix} \underline{\underline{A}}_{g,C}^{C,C} & \underline{\underline{A}}_{g,C}^{C,I} & \underline{\underline{0}} \\ \underline{\underline{A}}_{g,C}^{I,C} & \underline{\underline{A}}_{g}^{I,I} & \underline{\underline{A}}_{g,R}^{I,R} \\ \underline{\underline{0}} & \underline{\underline{A}}_{g,R}^{R,I} & \underline{\underline{A}}_{g,R}^{R,R} \end{bmatrix} \begin{bmatrix} \underline{u}_{g}^{C} \\ \underline{u}_{g}^{I} \\ \underline{u}_{g}^{R} \end{bmatrix} = \begin{bmatrix} \underline{s}_{g,C}^{C} \\ \underline{s}_{g}^{I} \\ \underline{s}_{g,R}^{R} \end{bmatrix} \tag{33}$$

where:

$$\underline{\underline{A}}_{g}^{I,I} = \underline{\underline{A}}_{g,C}^{I,I} + \underline{\underline{A}}_{g,R}^{I,I} \tag{34}$$

$$\underline{s}_{g}^{I} = \underline{s}_{g,C}^{I} + \underline{s}_{g,R}^{R} \tag{35}$$

The solution of (33) gives the internal flux distribution of the core and the flux/current distribution at the boundaries of the reflector. The flux at any point within the reflector could be evaluated simply by taking $\vec{\rho}$ as an internal point in the reflector equation (12). The details about this have been elaborated elsewhere [7].

3. Applications

The newly extended FE/BE hybrid method has been implemented in the FORTRAN program MGNEDPCM. The program is capable of handling multigroup diffusion theory problems of the k_{eff} determination type for reflected reactors.

The first problem we consider is a three-group treatment of a square core of side length a=70cm surrounded by a reflector of thickness b=10cm from the left and right. There is no reflector at the top or at the bottom. Due to the symmetry, only the upper-right quadrant of the system is discretized. Reflection boundary conditions on the left and bottom sides of the upper right quadrant are utilized to impose the symmetry of the full system. Naturally, vacuum boundary conditions prevail at the top and right sides of the upper quadrant to be discretized. The three group constants are presented in Table 1. There is direct coupling in the group-to-group scattering.

This problem has been run with our hybrid FEM/BEM program MGNEDPCM using a basic mesh. The basic hybrid mesh consists of 16x16 bilinear (rectangular) FEM mesh in the core. The hybrid mesh in the reflector consists of a basic BEM mesh with 3 linear elements per side on the shorter sides, 16 linear elements per side on the longer side. This basic hybrid mesh is said to have a degree of refinement n=1. A mesh which consists of a 16nx16n core FEM mesh and a 3n linear element per shorter side-16n linear element per longer side reflector BEM mesh is said to have a degree of refinement of n.

	Group	D_g(cm)	$\Sigma_{r,g}$(cm^{-1})	$\upsilon\Sigma_{f,g}$(cm^{-1})	$\Sigma_{s,g+1\leftarrow,g}$(cm^{-1})	χ_g
CORE	1	1.664	0.04595	0.00407154	0.04239	1.
	2	0.648	0.090624	0.00848	0.0676	0.
	3	0.3512	0.1261	0.181	-	0.
REFLECTOR	1	1.974	0.0733	0.	0.07327	-
	2	0.577	0.1501	0.	0.1501	-
	3	0.16	0.0197	0.	-	-

Table 1 Three group cross sections

The same problem has also been solved with computer programs implementing pure FEM and BEM. The pure FEM program we have used is FEND [15]. The basic FEM mesh of FEND entails a 16x16 bilinear (rectangular) element mesh in the core and a 3x16 bilinear element mesh in the reflector. This basic FEM mesh is said to have a degree of refinement of n=1. A 16nx16n core, 3nx16n reflector FEM mesh is said to have a degree of refinement of n. The pure BEM program we have used is GLOBAL (Ozgener and Ozgener, 2001). GLOBAL uses constant boundary elements. The basic BEM mesh involves 16 constant boundary elements per side in the core and 16 constant boundary elements/longer side and 3 constant boundary elements/shorter side in the reflector. A BEM mesh with 16n constant boundary elements per side in the core and 16n constant elements/longer side, 3n constant elements/shorter side in the reflector is said again to have a degree of refinement of n.

This problem has been run with our programs implementing the hybrid method, linear FEM and constant BEM (CBEM). In Table 2, the number of unknowns and the effective multiplication factors (k_{eff}) obtained for various degrees of refinements are presented. The FEM results with n=16 constitute the reference solution. The per cent errors in parenthesis are relative to the k_{eff} of the reference solution.

For a given degree refinement, the number of unknowns in the hybrid formulation is always less than the number of unknowns in FEM due to the boundary only discretization of the reflector in the hybrid method. For a given degree of refinement, the number of unknowns in the hybrid method is larger than the numbers of unknowns in CBEM which entails a boundary only discretization also for the core. For a given degree of refinement n, the hybrid method gives the smallest per cent error of the three methods. For higher degrees of refinement, the number of unknowns in the CBEM is much smaller than the other two methods and the accuracy in k_{eff} of CBEM is between the hybrid and the FEM. In Table 3 and Table 4, the average group fluxes for the core and the reflector regions are given. The fluxes are so normalized that the system average value of the total flux equals unity. The group g average flux is denoted by $\bar{\phi}_g \ (g = 1,2,3)$. In all three methods the errors associated with group average fluxes in the reflector are larger than their counterparts in the core. The hybrid method overestimates the reference group average fluxes both in the core and the reflector with the exception of the third group reflector average flux for which all three numerical solutions (Hybrid, FEM, CBEM) yield underestimates. FEM and CBEM give overestimates for the first and second energy groups in the reflector and underestimates in the core. The reverse is valid for the third group. All three numerical methods converge as the degree of refinement is increased.

	Number of unknowns/group			k_{eff}		
n	Hybrid	FEM	CFEM	Hybrid	FEM	CBEM
1	327	340	102	1.016975 (-0.0067%)	1.017591 (-0.0539%)	1.017364 (0.0316%)
2	1165	1287	204	1.017017 (-0.0025%)	1.017175 (0.0130%)	1.017112 (0.0068%)
4	4377	5005	408	1.017029 (-0.0013%)	1.017069 (0.0026%)	1.017052 (0.0009%)
16	-	78385	-	-	1.017036	-

Table 2 Number of unknowns and k_{eff} results for the three-group problem

	$\bar{\phi}_1$			$\bar{\phi}_2$			$\bar{\phi}_3$		
n	Hybrid	FEM	CBEM	Hybrid	FEM	CBEM	Hybrid	FEM	CBEM
1	0.69382 (0.280%)	0.69093 (-0.137%)	0.69132 (-0.081%)	0.31600 (0.281%)	0.31471 (-0.130%)	0.31468 (-0.141%)	0.17302 (0.322%)	0.17285 (0.223%)	0.17318 (0.413%)
2	0.69239 (0.074%)	0.69165 (-0.033%)	0.69174 (-0.020%)	0.31535 (0.074%)	0.31502 (-0.031%)	0.31501 (-0.034%)	0.17260 (0.079%)	0.17256 (0.054%)	0.17264 (0.104%)
4	0.69202 (0.020%)	0.69183 (-0.007%)	0.69186 (-0.003%)	0.31518 (0.020%)	0.31510 (-0.006%)	0.31510 (-0.007%)	0.17250 (0.017%)	0.17249 (0.011%)	0.17251 (0.024%)
16	-	0.69188	-	-	0.31512	-	-	0.17247	-

Table 3 Core group average fluxes for the three-group problem

	$\bar{\phi}_1$			$\bar{\phi}_2$			$\bar{\phi}_3$		
n	Hybrid	FEM	CBEM	Hybrid	FEM	CBEM	Hybrid	FEM	CBEM
1	0.10284 (4.786%)	0.10250 (4.438%)	0.10587 (7.875%)	0.05234 (2.636%)	0.05288 (3.696%)	0.05385 (5.592%)	0.20487 (-8.021%)	0.21990 (-1.271%)	0.21317 (-4.292%)
2	0.09928 (1.157%)	0.09918 (1.052%)	0.10007 (1.950%)	0.05131 (0.617%)	0.05144 (0.868%)	0.05170 (1.380%)	0.21821 (-2.032%)	0.22207 (-0.298%)	0.22035 (-1.072%)
4	0.09838 (0.237%)	0.09835 (0.210%)	0.09858 (0.440%)	0.05105 (0.111%)	0.05108 (0.173%)	0.05116 (0.306%)	0.22162 (-0.498%)	0.22260 (-0.059%)	0.22216 (-0.258%)
16	-	0.09814	-	-	0.05100	-	-	0.22273	-

Table 4 Reflector group average fluxes for the three-group problem

The second problem, we consider involves a four-group reflected system. The upper two groups are in the fast energy range and there is direct coupling in group-to-group scattering. The reflected system consists of a square core with a sidelength of 45 cm surrounded by a reflector of thickness 15 cm from the left and the right. Again there is no reflector at the top or bottom. Due to the symmetry, only the upper right quadrant is discretized. Reflective boundary conditions on the left and bottom sides of the upper-right quadrant are utilized to impose the symmetry of the full system. Vacuum boundary conditions prevail at the top and right sides of the upper-right quadrant. The four group constants for the core and the reflector are presented in Table 5.

	Group	D_g(cm)	$\Sigma_{r,g}$ (cm^{-1})	$\upsilon\Sigma_{f,g}$ (cm^{-1})	$\Sigma_{s,g+1\leftarrow g}$ (cm^{-1})	χ_g
CORE	1	2.1623	0.08795	0.009572	0.083004	0.575
	2	1.0867	0.06124	0.001193	0.0584	0.425
	3	0.6318	0.09506	0.01768	0.0.06453	0.0
	4	0.3543	0.121	0.18514	-	0.0
REFLECTOR	1	3.2306002	0.094135	0.0	0.094135	-
	2	0.9448411	0.13534	0.0	0.13534	-
	3	0.6012235	0.139863	0.0	0.13869	-
	4	0.1450474	0.019095	0.0	-	-

Table 5 Four group cross sections for the core and the reflector

The notation for the meshes used for the hybrid, FEM and CBEM solutions are similar to the first problem and involve a parameter n, which denotes the degree of refinement. The hybrid mesh with a degree of refinement n involves a 5nx5n bilinear (rectangular) element FEM mesh in the core and a 5n linear element per side BEM mesh in the reflector. The FEM solution with a degree of refinement n involves a 5nx5n bilinear (rectangular) element finite element meshes in both the core and the reflector. The CBEM solution with a degree of refinement n involves 5n constant element per side BEM mesh both in the core and the reflector.

The number of unknowns per energy group, the effective multiplication factor (keff) and per cent error in keff relative to the reference (n=32, FEM) solution are presented in Table 6. The * indicates that particular hybrid run is made with a 4x4 core FEM and a 5 element/side reflector BEM mesh.

	Number of unknowns/group			k_{eff}		
n	Hybrid	FEM	BEM	Hybrid	FEM	BEM
1	56	66	40	0.937712 (-0.8355%)	0.945779 (0.0177%)	0.950155 (0.4804%)
2	161	231	80	0.945258 (-0.0374%)	0.945652 (0.0042%)	0.946871 (0.1331%)
4	521	861	160	0.945577 (-0.0100%)	0.945621 (0.0010%)	0.945932 (0.0338%)
32	-	51681	-	-	0.945612	

Table 6 Number of unknowns and keff results for the four group problem

As in the three group problem, FEM and CBEM approach the reference k_{eff}. from above, while the hybrid method approaches from below as n is increased. Of the three methods, the FEM solution gives the smallest per cent errors in k_{eff} for a given degree of refinement. This is in contrast to the three group problem where the hybrid method among the three methods produced the best k_{eff} values for a specified value of n. All three methods converge to the reference k_{eff} value as the meshes are refined. In Fig. 2 and 3, the average core and reflector group fluxes are plotted as a function of n, the degree of refinement. The fluxes are again so normalized that the system average value of the total flux equals unity. The reference solution (n=32, FEM) is indicated by the dashed horizontal lines. A study of the Fig. 2 and 3 indicates that again FEM yields the best group average fluxes among the three methods. Nevertheless, the hybrid method is a close second.

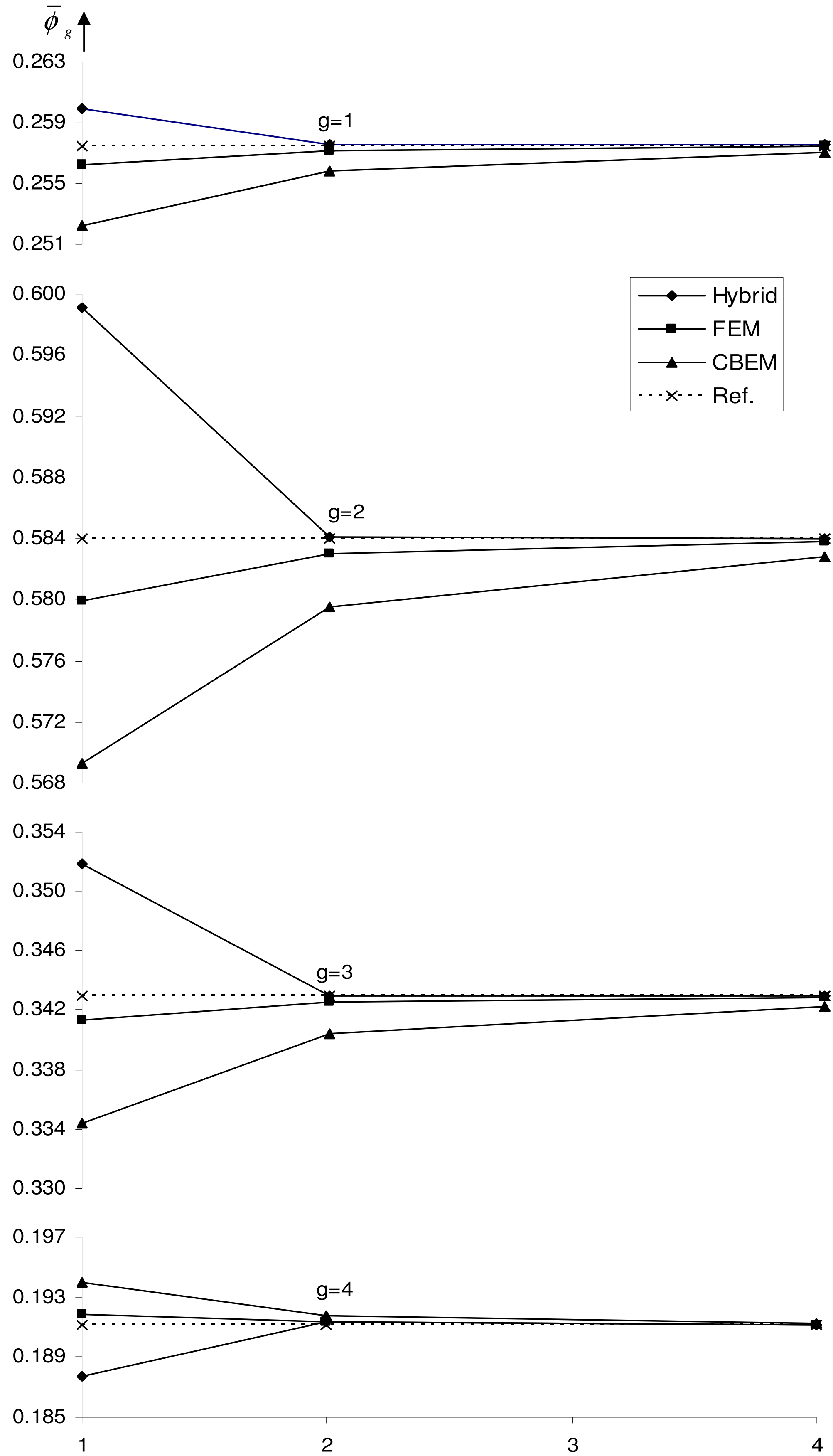

Figure 2 Group core avrage fluxes

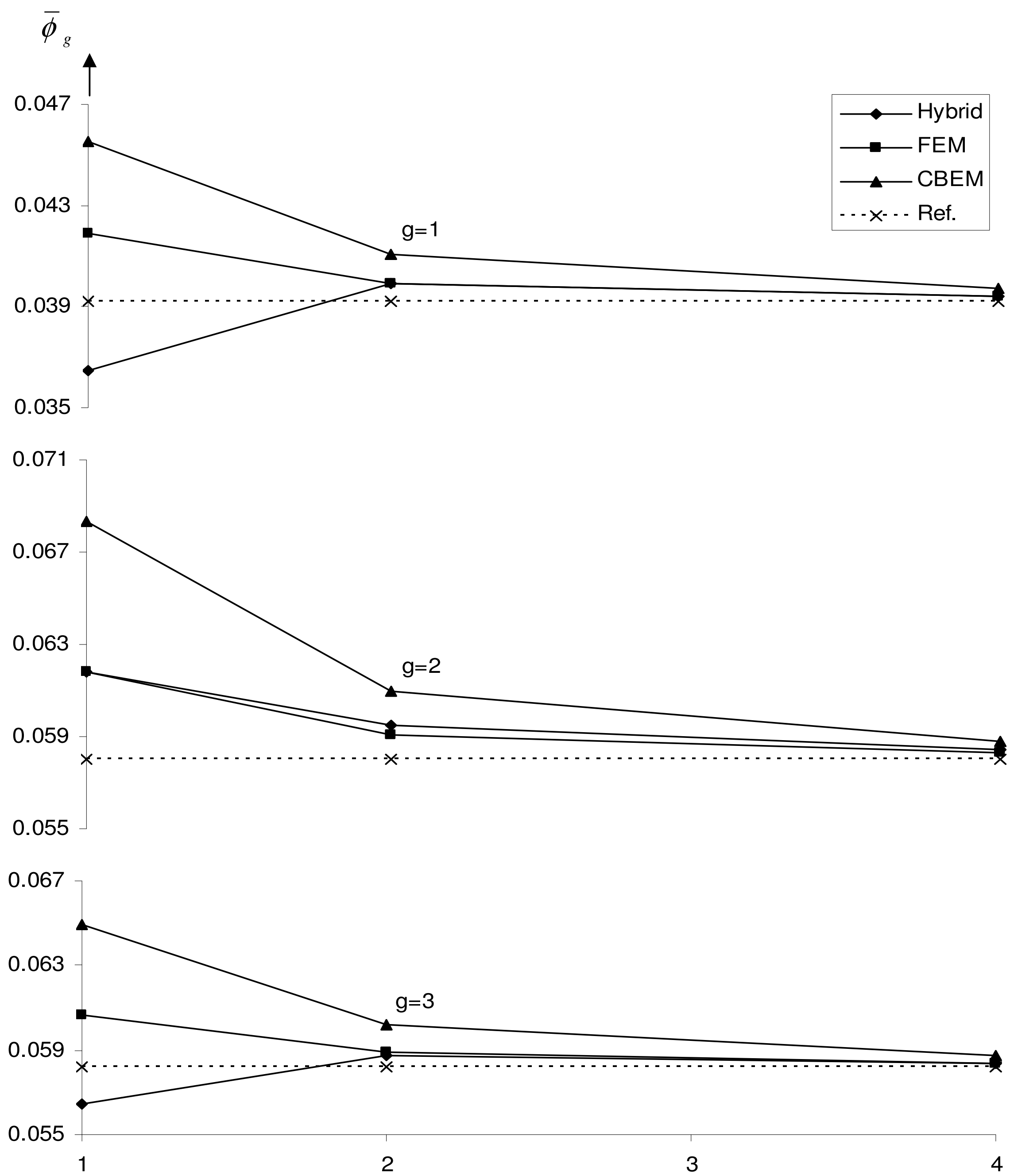

Figure 3 Group reflector average fluxes for the first three group

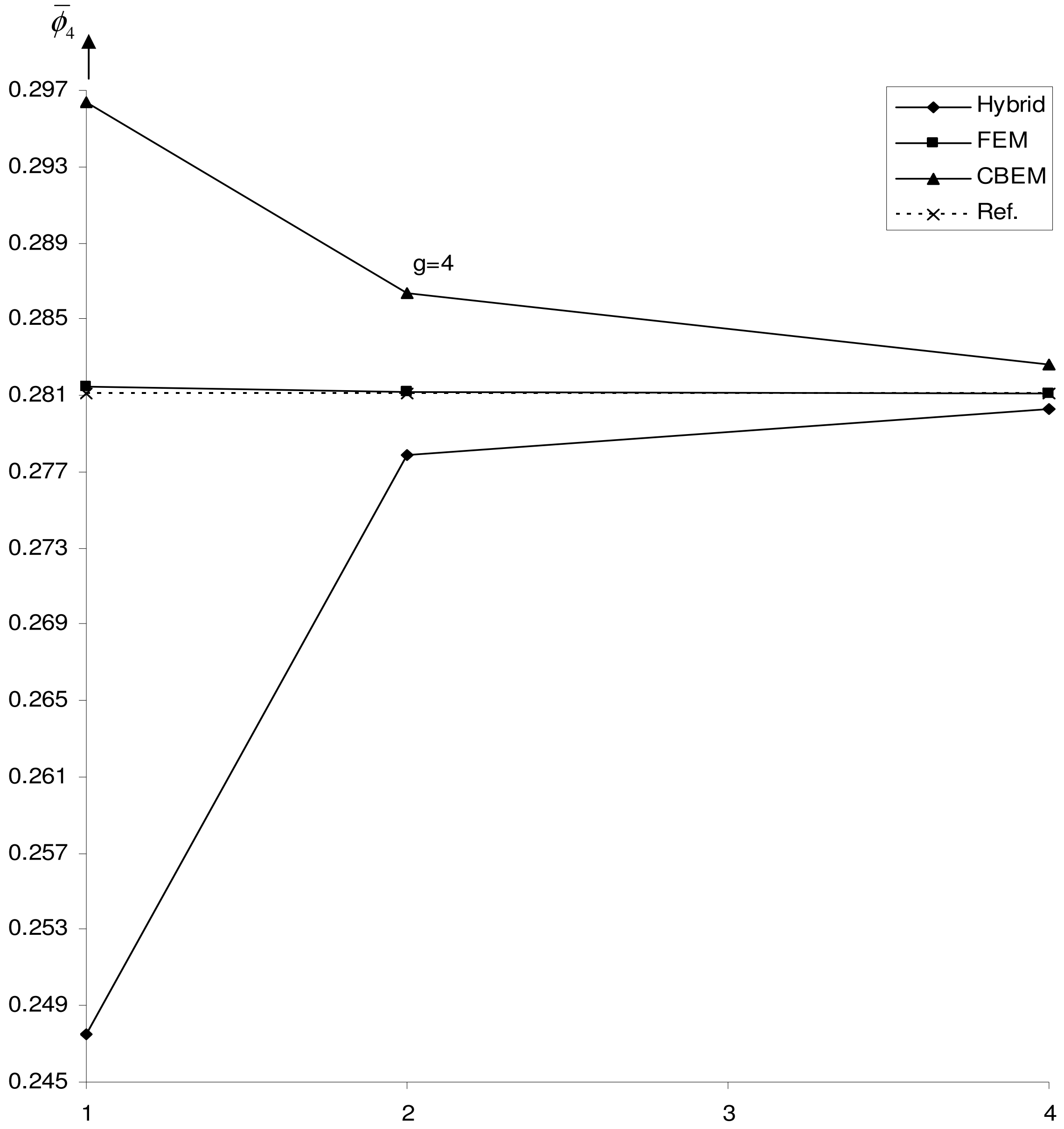

Figure 4 Fourth group reflector average fluxes

4. Conclusions and recommendations

In this work, the FEM/BEM hybrid method for reflected reactors have been extended to general multigroup diffusion theory problems. The comparisons with the constant BEM and linear FEM solutions within the context of three and four group theories indicate that the hybrid method is capable of producing accurate results giving the best (three group problem) or a close second best (four group problem) results among the three methods. Thus further research in this area seems to be warranted. A general numerically optimized multiregion formulation with an option of using FEM or BEM in each region seems to be a good choice as a topic for further research.

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