Adomian Decomposition Method For Neutron Diffusion Calculations

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Adomian Decomposition Method – 1

- Solution in the form of a series expansion.
- Can be applied to
  - Linear/non-linear ordinary/partial differantial equations,
  - Integro-differential equations,
  - Integral or differential delay equations,
- No need to modify the original problem or perform linearizations.
- The resulting computation schemes are efficient, with high accuracy, and generaly a rapidly convergent series solution is achieved

Adomian Decomposition Method - 2

L invertible linear operator

$$Fu = g$$

$$Fu = Lu + Ru = g$$

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru$$

$$u = A + Bx + L^{-1}g - L^{-1}Ru$$

$$u = \sum_{n=0}^{\infty} u_n = u_0 - L^{-1}R\sum_{n=0}^{\infty} u_n$$

$$u_0 = A + Bx + L^{-1}g$$

$$u_1 = -L^{-1}Ru_0$$

$$u_2 = -L^{-1}Ru_1$$

$$\vdots$$

$$u_{n+1} = -L^{-1}Ru_n$$

Solution of NDE through ADM - 1

$$\frac{\partial^2\varphi(x,y)}{\partial x^2}+\frac{\partial^2\varphi(x,y)}{\partial y^2}-\chi^2\varphi(x,y)=-\frac{S}{D}$$

$$x=0 \quad \frac{\partial\varphi(x,y)}{\partial x}=0 \qquad x=a \quad \varphi(a,y)=0$$

$$y=0 \quad \frac{\partial\varphi(x,y)}{\partial y}=0 \qquad y=a \quad \varphi(x,a)=0$$

$$L_x=\frac{\partial^2}{\partial x^2},\ L_y=\frac{\partial^2}{\partial y^2}$$

$$L_x^{-1}=\iint .\,dxdx$$

$$L_x\varphi(x,y)+L_y\varphi(x,y)-\chi^2\varphi(x,y)=-\frac{S}{D}$$

$$L_x^{-1}L_x\varphi(x,y)=L_x^{-1}(\chi^2\varphi(x,y))+L_x^{-1}L_y\varphi(x,y)-L_x^{-1}\left(\frac{S}{D}\right)$$

Solution of NDE through ADM - 2

$$L_x^{-1} L_x \varphi(x,y) = \varphi(x,y) + A(y)x + B(y)$$

$$\varphi(x,y) = A(y)x + B(y) + L_x^{-1}(\chi^2 \varphi(x,y)) + L_x^{-1} L_y \varphi(x,y) - \frac{S}{D}\frac{x^2}{2}$$

The ADM iterations take the following form;

$$\varphi_0 = B(y) - \frac{S}{D}\frac{x^2}{2}$$

$$\varphi_1 = L_x^{-1}(\chi^2 \varphi_0) - L_x^{-1} L_y \varphi_0$$

$$\varphi_2 = L_x^{-1}(\chi^2 \varphi_1) - L_x^{-1} L_y \varphi_1$$

$$\vdots$$

$$\varphi_n = L_x^{-1}(\chi^2 \varphi_{n-1}) - L_x^{-1} L_y \varphi_{n-1}$$

Solution of NDE through ADM - 3

When we apply first boundary condition at x=0;

$$A(y) = 0 \qquad \varphi_0 = B(y) - \frac{S}{D}\frac{x^2}{2}$$

$$B(y) = \sum_{m=0}^{\infty} b_m \, Cos(\beta_m y)$$

$$\varphi_1 = L_x^{-1}(\chi^2 \varphi_0) - L_x^{-1} L_y \varphi_0$$

$$\beta_m = \frac{(2m+1)\pi}{2a}$$

$$\frac{S}{D}\frac{x^2}{2} = \sum_{m=0}^{\infty} f_m \, Cos(\beta_m y) \longrightarrow f_m = (-1)^m \frac{2S}{D\beta_m a}\frac{x^2}{2}$$

$$\varphi_0 = \sum_{m=0}^{\infty} b_m \, Cos(\beta_m y) - \frac{2S}{Da}\frac{x^2}{2}\sum_{m=0}^{\infty}\frac{(-1)^m}{\beta_m} Cos(\beta_m y)$$

Solution of NDE through ADM - 4

$$\varphi_1 = L_x^{-1}(\chi^2\varphi_0) - L_x^{-1}L_y\varphi_0$$

$$\varphi_2 = L_x^{-1}(\chi^2\varphi_1) - L_x^{-1}L_y\varphi_1$$

$$\vdots$$

$$\varphi_n = L_x^{-1}(\chi^2\varphi_{n-1}) - L_x^{-1}L_y\varphi_{n-1}$$

$$\varphi_n = \sum_{m=0}^{\infty} b_m\, Cos(\beta_m y)\left(\frac{\alpha_m^{2n}x^{2n}}{(2n)!}\right) - \frac{2S}{Da}\sum_{m=0}^{\infty}\frac{(-1)^m}{\beta_m\alpha_m^2}Cos(\beta_m y)\left(\frac{\alpha_m^{2n}x^{2n+2}}{(2n+2)!}\right)$$

$\Rightarrow$

$$\varphi(x,y) = \sum_{n=0}^{\infty}\varphi_n = \frac{2S}{Da}\sum_{m=0}^{\infty}\frac{(-1)^m}{\beta_m\alpha_m^2}Cos(\beta_m y)\left[1 - \frac{Cosh(\alpha_m x)}{Cosh(\alpha_m a)}\right]$$

Solutions – $\varphi(x,y)$

$$\beta_m = \frac{(2m+1)\pi}{2a}$$

$$\alpha_m^2 = \chi^2 + \beta_m^2$$

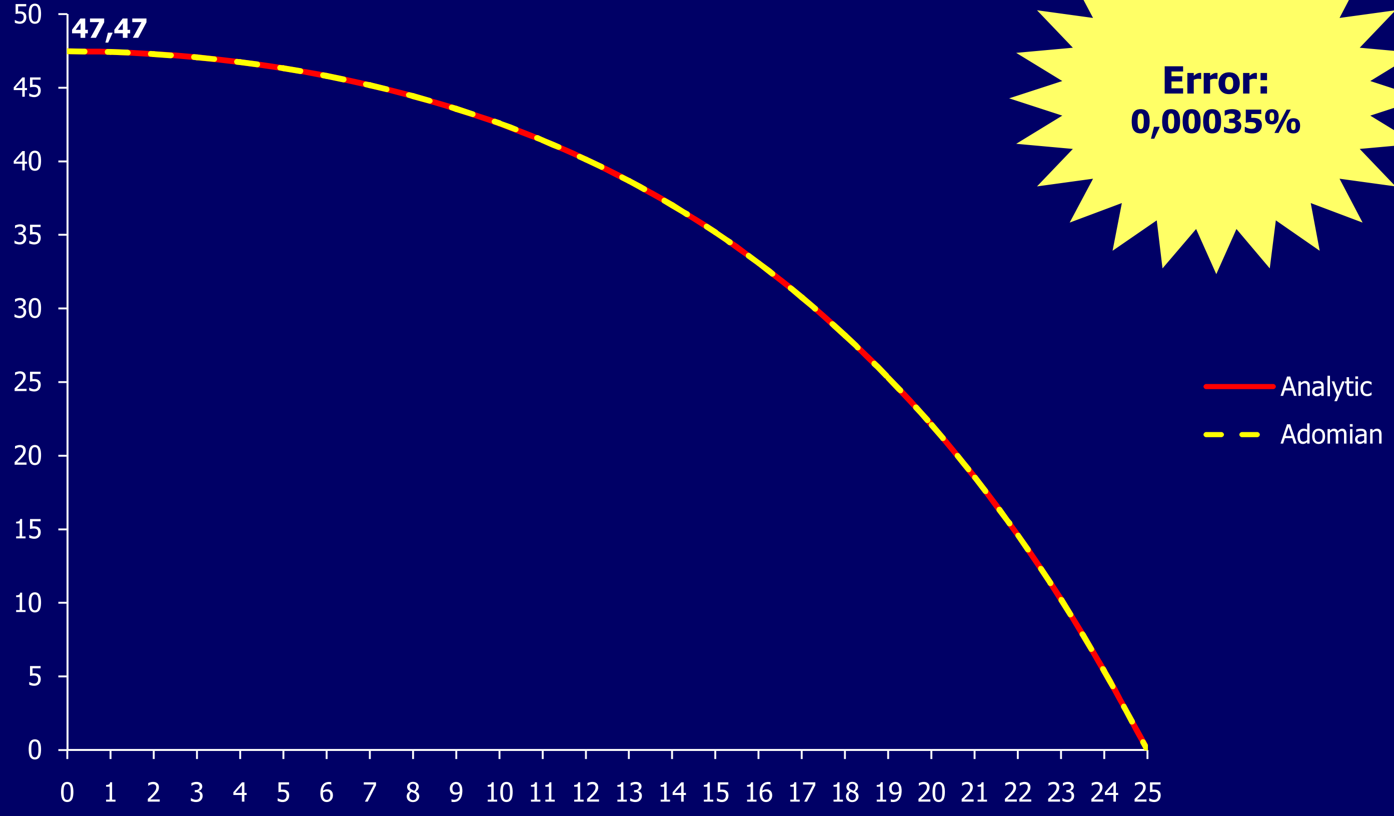
- Analytical Solution

$$\varphi(x,y) = -\frac{2S}{Da}\sum_{m=0}^{\infty}\frac{(-1)^m}{\beta_m\alpha_m^2}\frac{Cosh(\alpha_m x)}{Cosh(\alpha_m a)}Cos(\beta_m y) + \frac{S}{D\chi^2}\left(1-\frac{Cosh(\chi y)}{Cosh(\chi a)}\right)$$

- Adomian Decomposition Method's Solution

$$\varphi(x,y) = \frac{2S}{Da}\sum_{m=0}^{\infty}\frac{(-1)^m}{\beta_m\alpha_m^2}Cos(\beta_m y)\left[1-\frac{Cosh(\alpha_m x)}{Cosh(\alpha_m a)}\right]$$

Solutions – Graphics of $\varphi(x,0)$



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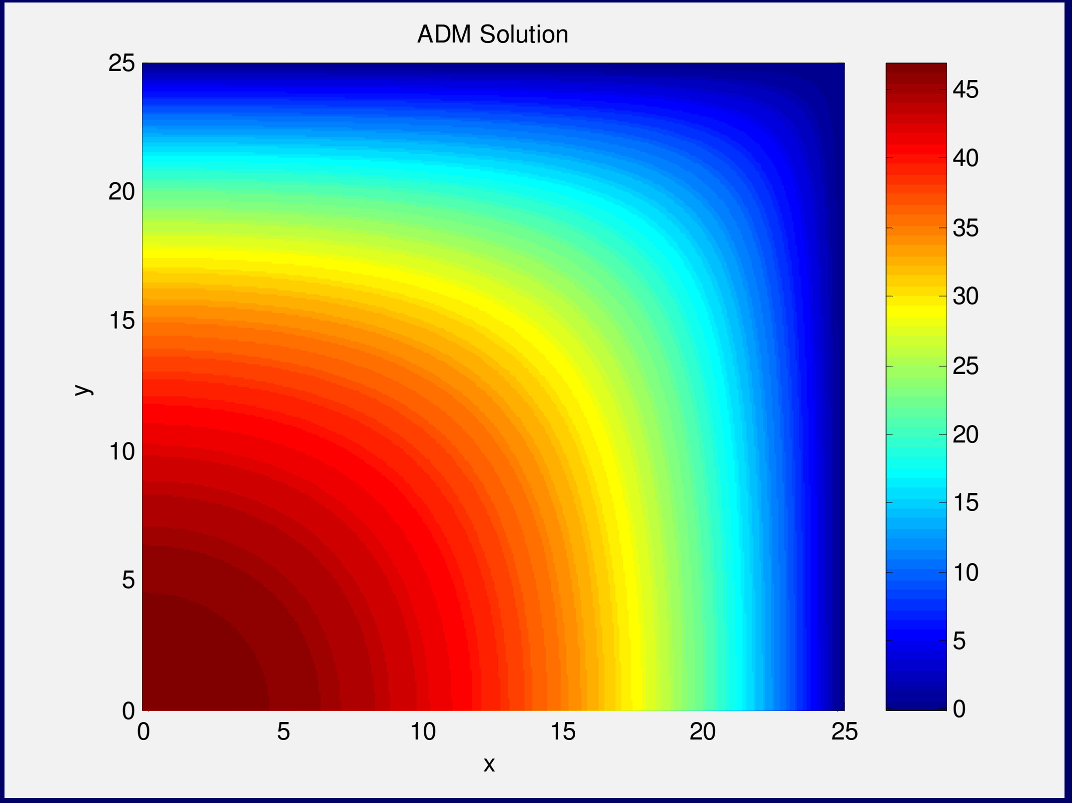
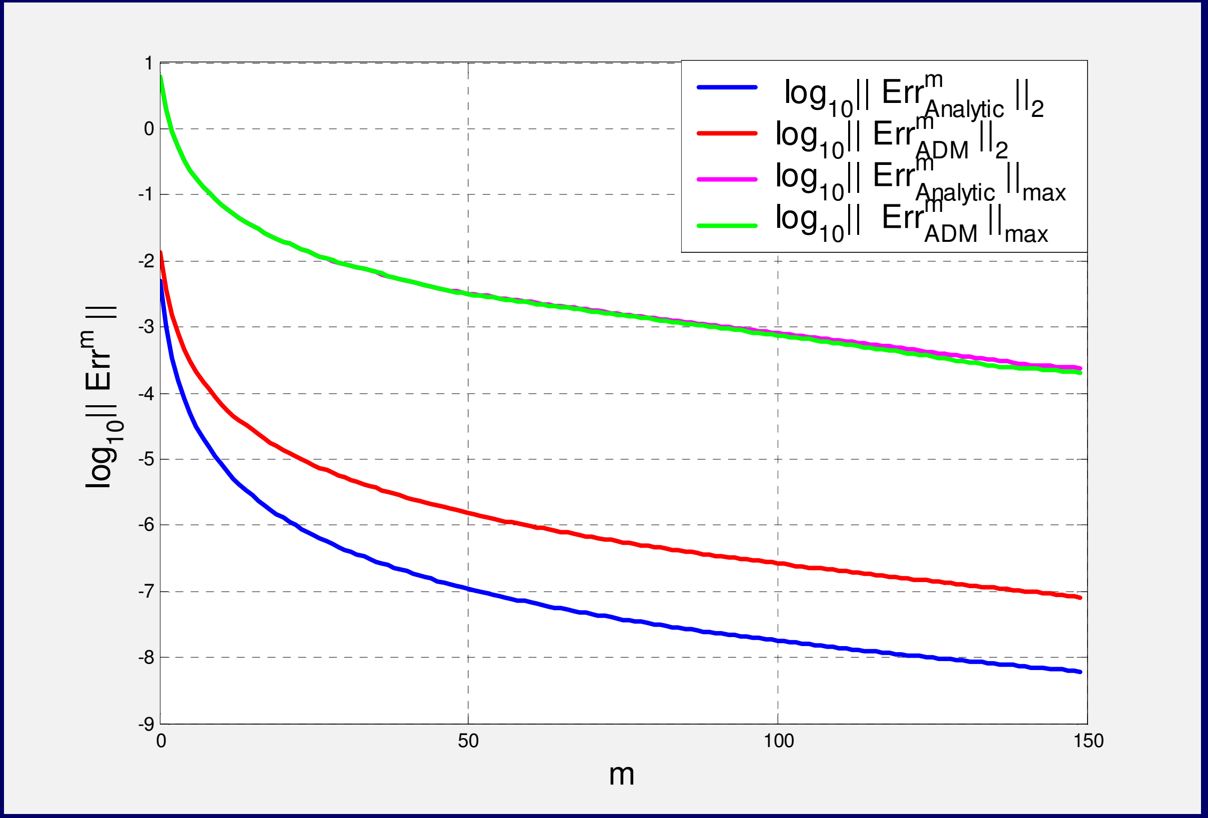
Solutions – Graphics of $\varphi(x, y)$



Solutions – Error



Conclusion

- ADM solution converges to the Analytic solution (obtained by splitting and separation of variables).
- Low overhead; it is easy to infer the general term of the series expansion after computing the first three elements of the ADM iterations.
- Promising for the problems where the forcing term is in a general form, i.e. g(x,y).